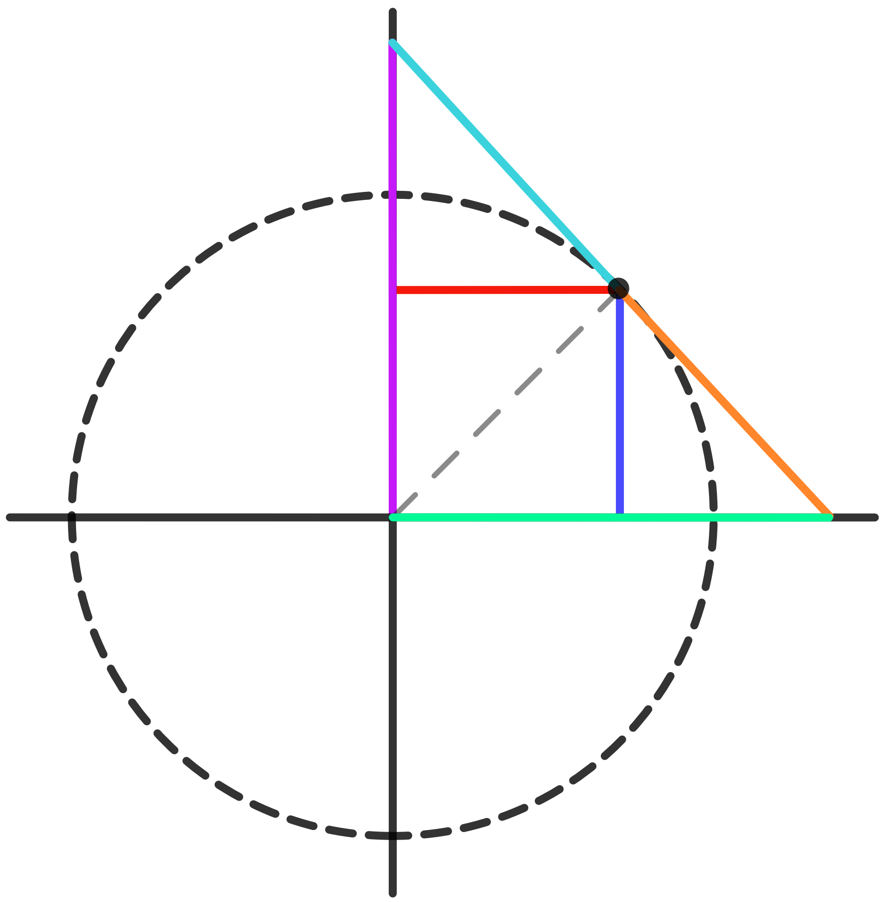
TRIG-OH-NAW-METRY

Part 1

Sine Cosine Tangent Cotangent Secant Cosecant



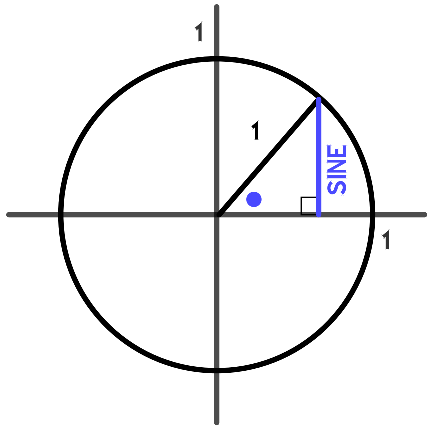
Line Trig

Sine Lengths and the Unit Circle

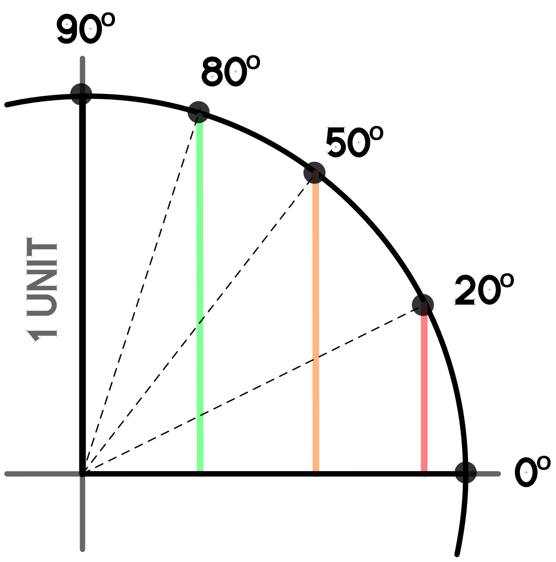
A 'Unit Triangle' is a right angled triangle with a hypotenuse of unit length.

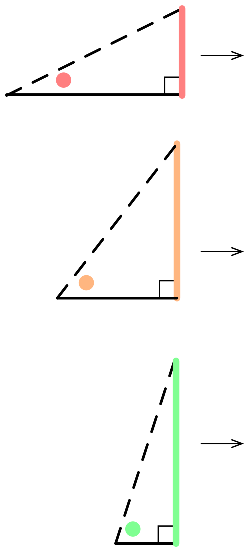
The height of this Unit Triangle is called the SINE.

The length of the SINE changes according to the angle of the Unit Triangle that is locted at the centre of the Unit Circle.



Mathematicians, like us, have very kindly recorded every sine length for every degree of rotation around the Unit Circle. All lengths can be found in the table below...





Natural Sine Table

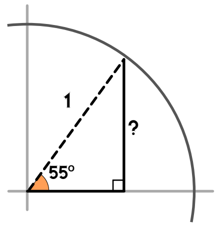
Angle	Sine
0	0.0
10	0.174
20	0.342
30	0.5
40	0.643
50	0.766
60	0.866
70	0.940
80	0.985
90	1.0

To find the sine length of a Unit Triangle we have to follow these two steps...

- Measure the angle opposite to the sine.
- Look to the natural sine table to find the corresponding length for that angle.

It Must be a SINE !

Task 1 – Use the table to find the missing lengths that are opposite the angles ...

1


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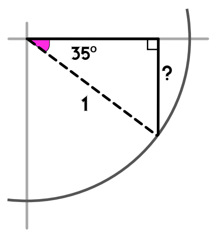

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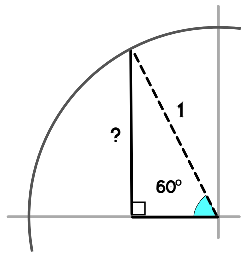

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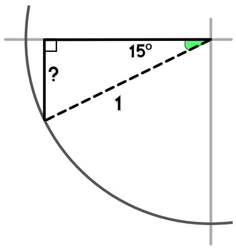

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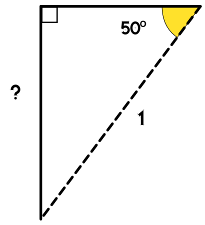

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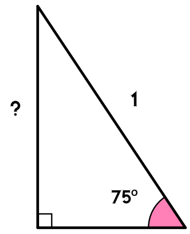

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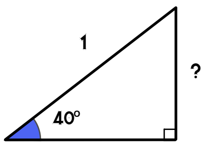

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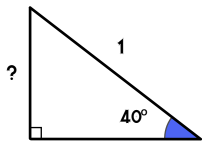

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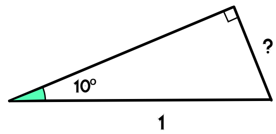
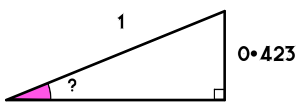


Task 2 – Find the missing lengths and angles...

1


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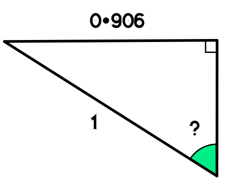

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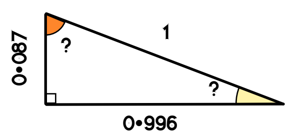

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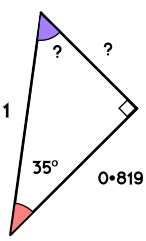

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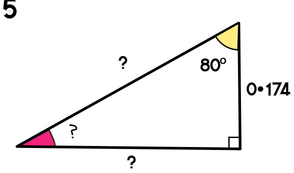

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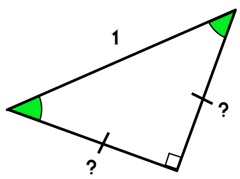

Natural Sine Table

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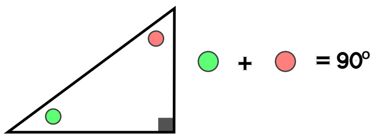
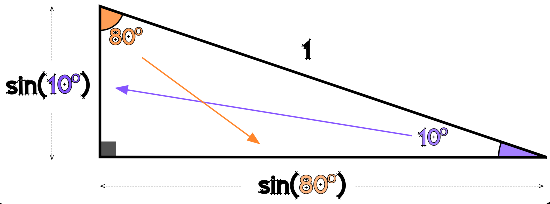
55	0.819
60	0.866
65	0.906
70	0.940
75	0.966
80	0.985
85	0.996
90	1.0

Line Trig

Angle Fact

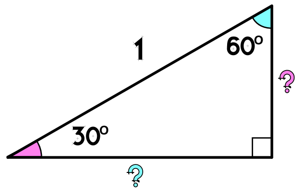


Which Sine is Which



Task 3 – Using the angel fact above, find all the missing lengths and angles...

1



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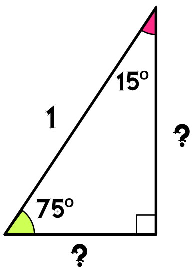

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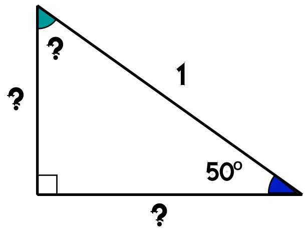

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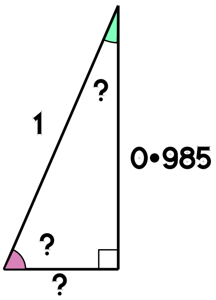

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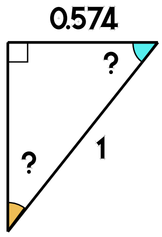

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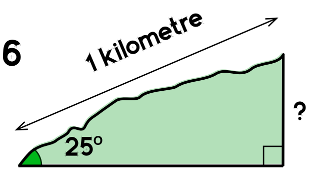
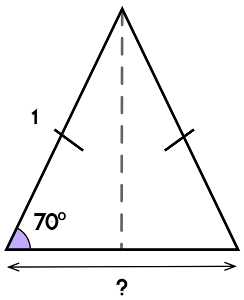


Jim is climbing a hill.
How high off the ground is Jim from where he started?
Answer in metres.

7



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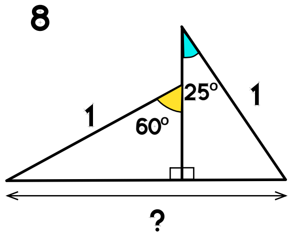

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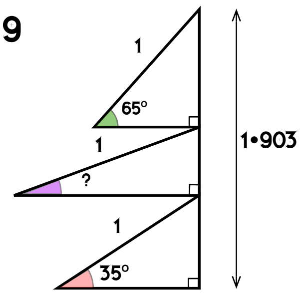

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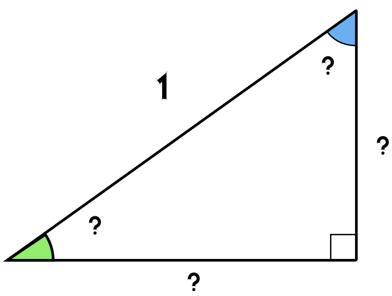
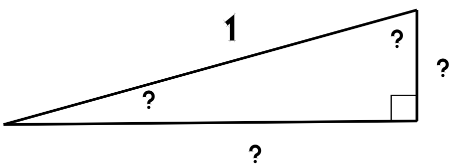
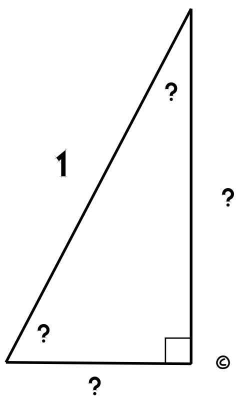
40	0.643
45	0.707
50	0.766
55	0.819
60	0.866
65	0.906
70	0.940
75	0.966
80	0.985
85	0.996
90	1.0

Task 4 – Fill in the blanks with possibe angles...

What do you notice about the relationship between the angles and lengths of the triangles ?

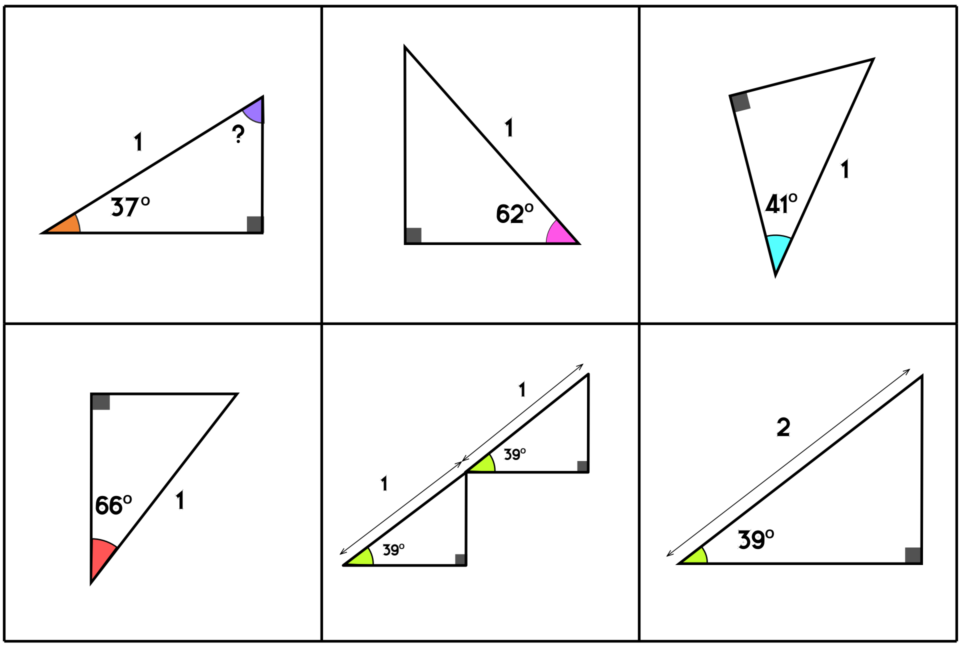






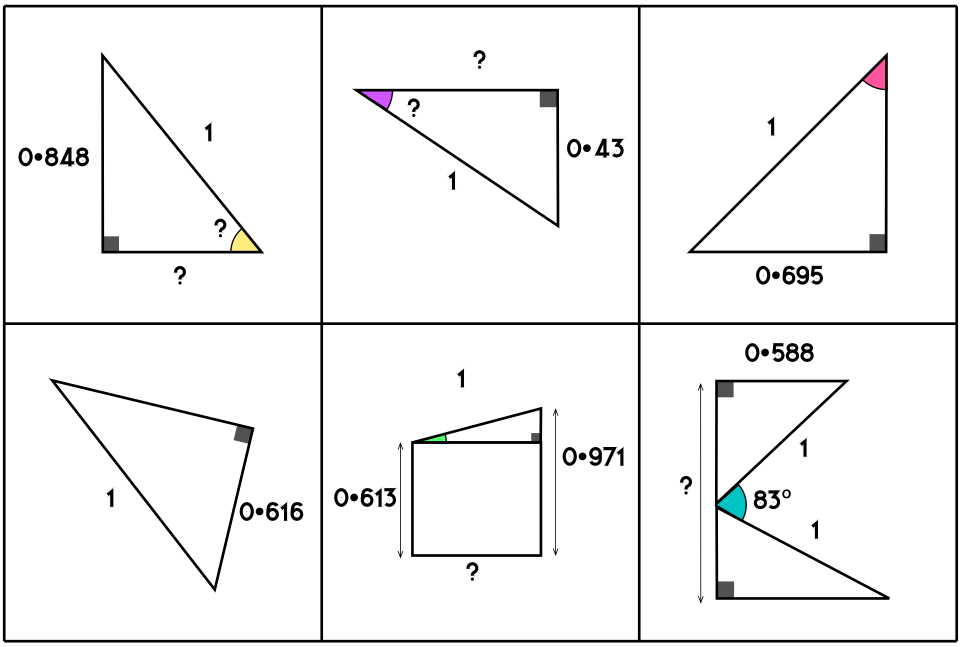
Sine Language

Task 1 – Find all the missing lengths for each of the following triangles…



Did you spot a quick way of using the table to find your lengths ?… Why does this work?… Describe why the last triangle is different from the rest and how did you calculate the missing lengths?…

Task 2 – Find all the missing lengths for each of the following shape…



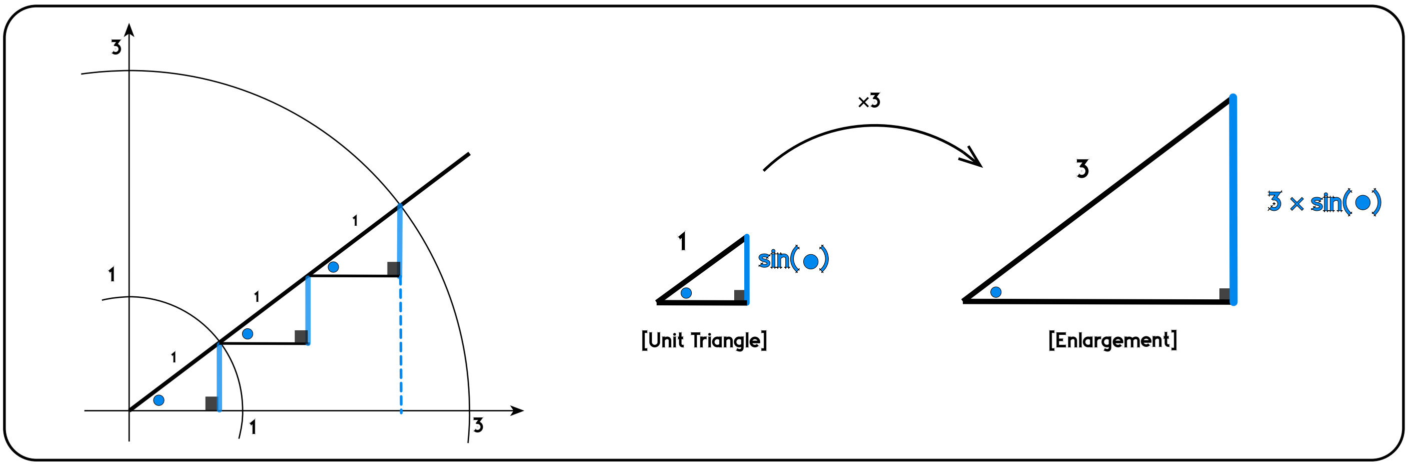
Angle	Sine		Angle
0	0.0	1.0	90
1	0.017	0.9998	89
2	0.035	0.9994	88
3	0.052	0.9986	87
4	0.070	0.9976	86
5	0.087	0.996	85
6	0.105	0.995	84
7	0.122	0.993	83
8	0.139	0.990	82
9	0.156	0.988	81
10	0.174	0.985	80
11	0.191	0.982	79
12	0.208	0.978	78
13	0.225	0.974	77
14	0.242	0.970	76
15	0.259	0.966	75
16	0.276	0.961	74
17	0.292	0.956	73
18	0.309	0.951	72
19	0.326	0.946	71
20	0.342	0.940	70
21	0.358	0.934	69
22	0.375	0.927	68
23	0.391	0.921	67
24	0.407	0.914	66
25	0.423	0.906	65
26	0.438	0.899	64
27	0.454	0.891	63
28	0.469	0.883	62
29	0.485	0.875	61
30	0.5	0.866	60
31	0.515	0.857	59
32	0.530	0.848	58
33	0.545	0.839	57
34	0.559	0.829	56
35	0.574	0.819	55
36	0.588	0.809	54
37	0.602	0.799	53
38	0.616	0.788	52
39	0.629	0.777	51
40	0.643	0.766	50
41	0.656	0.755	49
42	0.669	0.743	48
43	0.682	0.731	47
44	0.695	0.719	46
45	0.707	0.707	45

Beyond the Unit Circle

The Unit Circle has been useful for recording Sine values.

What if the Unit Circle is enlarged ?… Will the sides all be enlarged too ?… What about the angles?…

A fraction of the Unit Cirlce enlarged by a factor of 3…

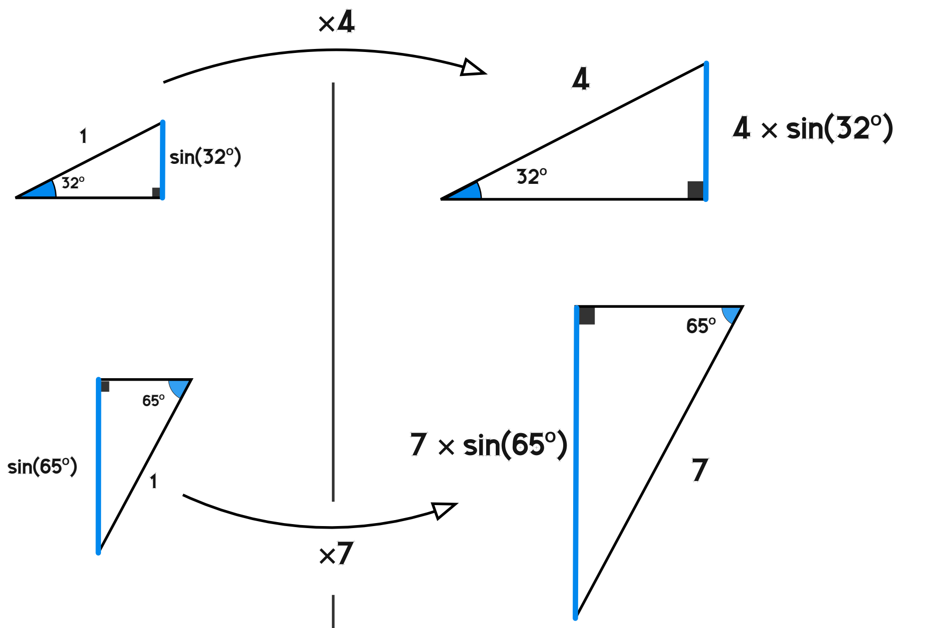


When the Unit Circle is enlarged by a scale factor two interesting things happen to the Unit Triangle.

The three sides of the triangle are enlarged by the same scale factor but the angles remain the same size!

Examples of Enlarging a Unit Triangle…

[Unit Triangles] [Enlargements] [Calculations]



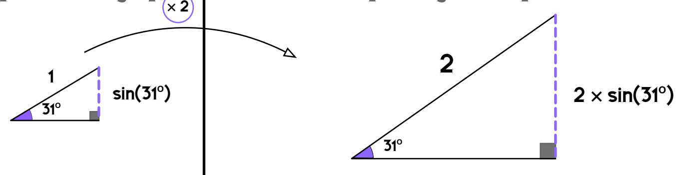
$4 \times 0{\bullet}530$
$= 2{\bullet}120$

$7 \times 0{\bullet}906$
$= 6{\bullet}342$

Enlarging Unit Triangles

Calculate the lengths of the lines opposite the given angles in the enlarged triangles...

[Unit Triangle] | [Enlargement] | [Calculation]

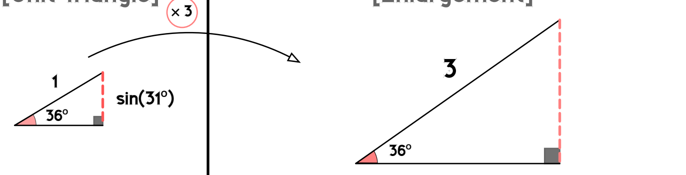


$2 \times \sin(31°)$

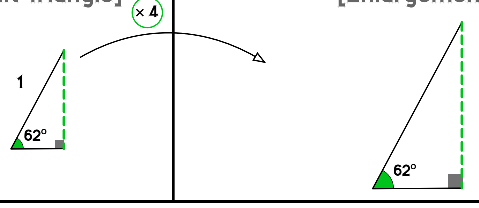
$= 2 \times$ ______

$=$ ______

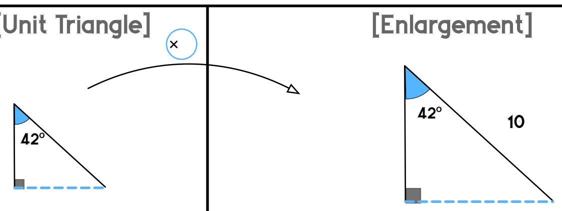
[Unit Triangle] | [Enlargement] | [Calculation]



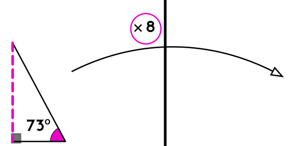
[Unit Triangle] | [Enlargement] | [Calculation]



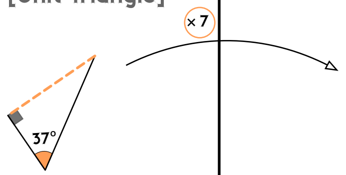
[Unit Triangle] | [Enlargement] | [Calculation]



[Unit Triangle] | [Enlargement] | [Calculation]



[Unit Triangle] | [Enlargement] | [Calculation]



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5	0.087	0.996	85
6	0.105	0.995	84
7	0.122	0.993	83
8	0.139	0.990	82
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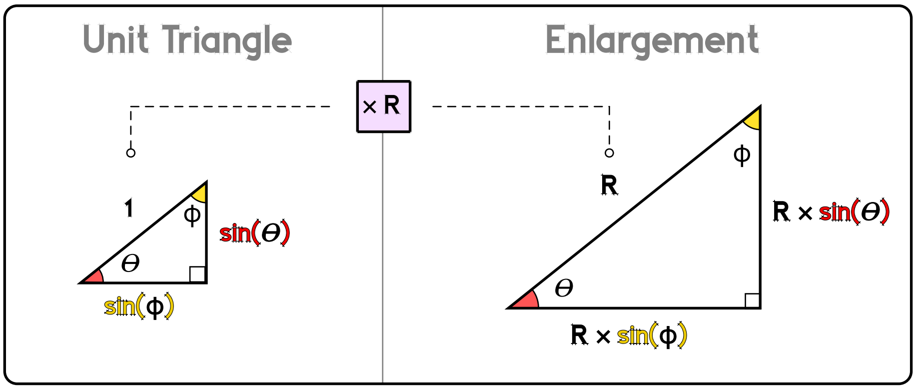
The General Case

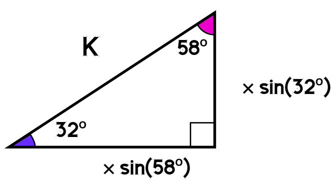


Observations…

…'If we know the value of the hypotenuse then we know the value of the enlargement scale factor. The other sides of the triangle are then multiplied by this value..'

Find every angle and label every side of these enlarged unit triangles …

1.



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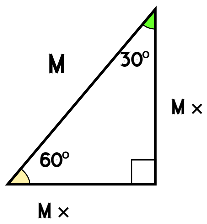

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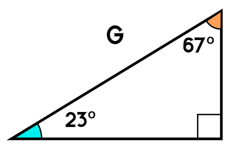

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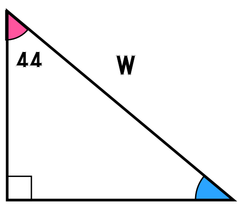

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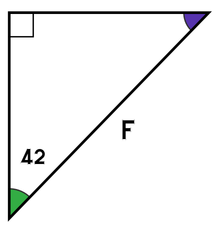

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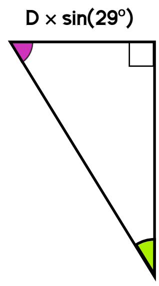

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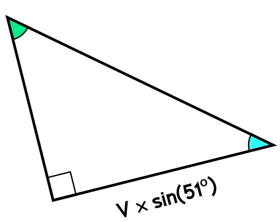
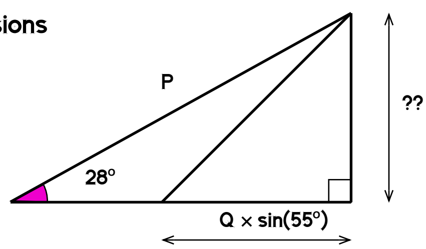


Extension

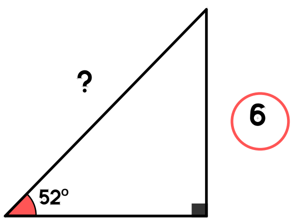
Find two different expressions for the missing length.



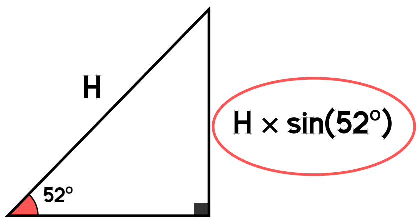
Calculating the Hypotenuse

Here we have two congruent (exactly the same) triangles…If we are not lucky enough to know the length of the hypotenuse, we can label it anything we like! I'll go for the letter H.
That means we can now label the side opposite the angle as well.

Unknown Hypotenuse



Labelled Hypotenuse



Working

$$H \times \sin(52^\circ) = 6$$

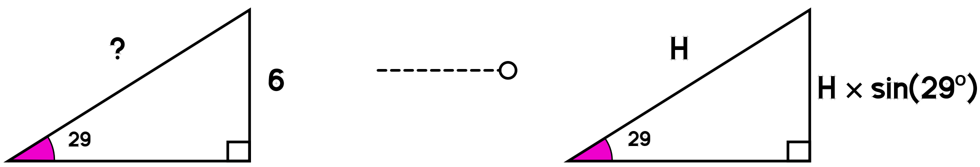
$$H = \frac{6}{\sin(52^\circ)}$$

$$H = \frac{6}{0{\cdot}788}$$

$$H = 7{\cdot}61$$

Calculate the missing length of the hypotenuse for the following right angled triangles…

Example 1



"is the hypotenuse greater than 6 ?"

Working

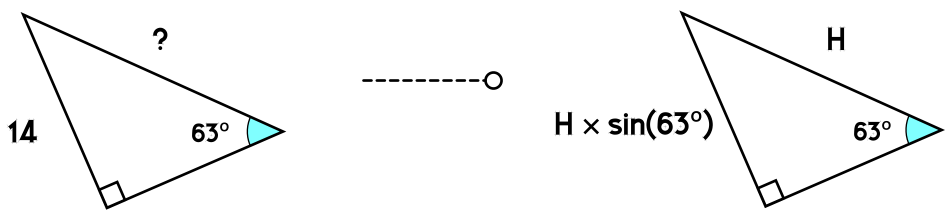
$$H \times \sin(29^\circ) = 6$$

$$H = \frac{6}{\sin(29^\circ)}$$

$$H = \frac{6}{0{\cdot}485}$$

$$H = \underline{\underline{12{\cdot}37}}$$

Example 2



Working

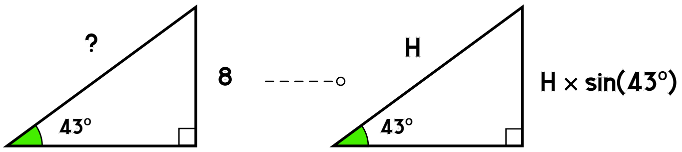
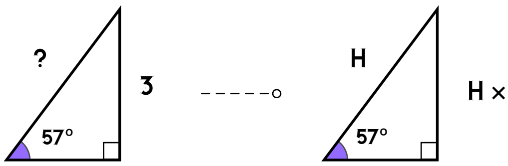
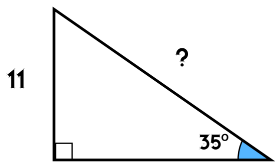
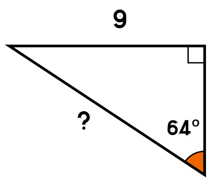
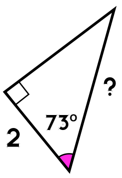
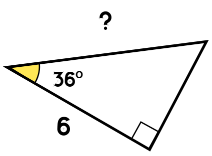
$$H \times \sin(63^\circ) = 14$$

$$H = \frac{14}{\sin(63^\circ)}$$

$$H = \frac{14}{0{\cdot}891}$$

$$H = \underline{\underline{15{\cdot}71}}$$

Working Backwards

Question	Working
	$H \times \sin(43°) = 8$ $H = \dfrac{8}{\sin(43°)} = \dfrac{8}{\square}$ $H =$
	
	
	
	
	

Angle	Sine		Angle
0	0.0	1.0	90
1	0.017	0.9998	89
2	0.035	0.9994	88
3	0.052	0.9986	87
4	0.070	0.9976	86
5	0.087	0.996	85
6	0.105	0.995	84
7	0.122	0.993	83
8	0.139	0.990	82
9	0.156	0.988	81
10	0.174	0.985	80
11	0.191	0.982	79
12	0.208	0.978	78
13	0.225	0.974	77
14	0.242	0.970	76
15	0.259	0.966	75
16	0.276	0.961	74
17	0.292	0.956	73
18	0.309	0.951	72
19	0.326	0.946	71
20	0.342	0.940	70
21	0.358	0.934	69
22	0.375	0.927	68
23	0.391	0.921	67
24	0.407	0.914	66
25	0.423	0.906	65
26	0.438	0.899	64
27	0.454	0.891	63
28	0.469	0.883	62
29	0.485	0.875	61
30	0.5	0.866	60
31	0.515	0.857	59
32	0.530	0.848	58
33	0.545	0.839	57
34	0.559	0.829	56
35	0.574	0.819	55
36	0.588	0.809	54
37	0.602	0.799	53
38	0.616	0.788	52
39	0.629	0.777	51
40	0.643	0.766	50
41	0.656	0.755	49
42	0.669	0.743	48
43	0.682	0.731	47
44	0.695	0.719	46
45	0.707	0.707	45

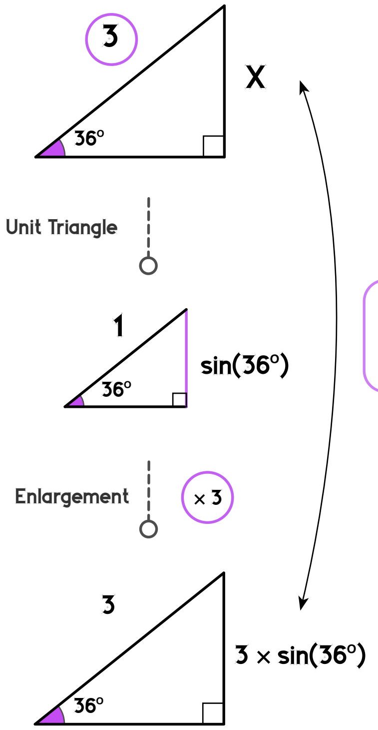
If in Doubt... Draw a Unit Triangle

When unsure how to choose which steps to take in order to calculate a missing length of a right angled triangle when you know one of the angles, dont panic!

There is always the option of drawing a unit triangle and then try to enlarge it back to the original triangle. Equations should POP out at you.

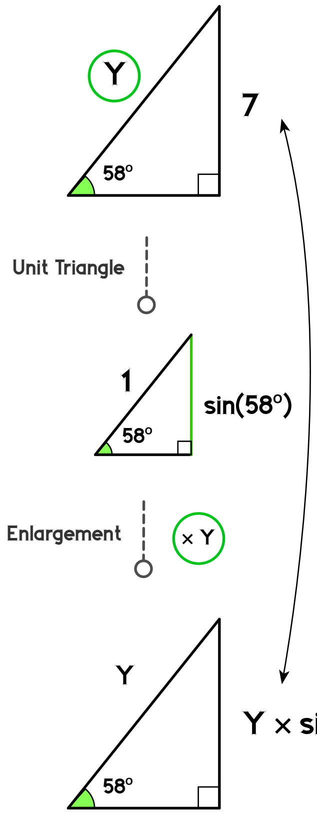
Finding an Equation to find the Missing Length in a Right Angled

Do we know the length of the hypotenuse?... YES!



$$X = 3 \times \sin(36^\circ)$$

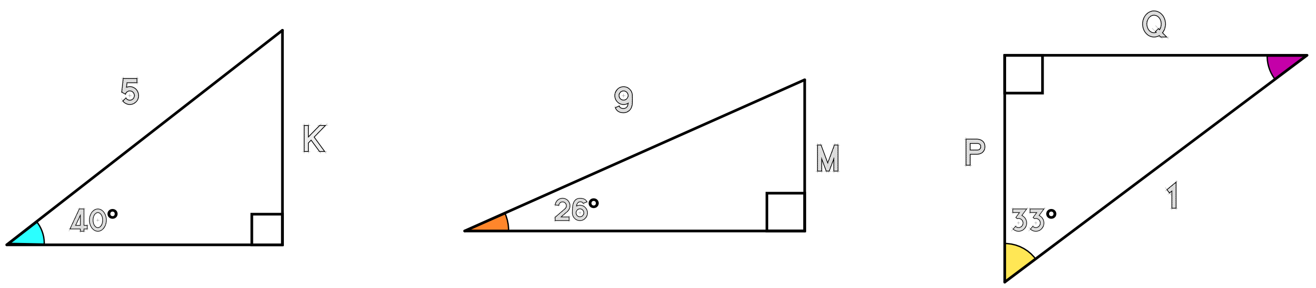
Do we know the length of the hypotenuse?... NO!



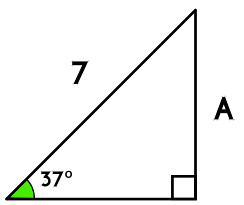
$$7 = Y \times \sin(58^\circ)$$

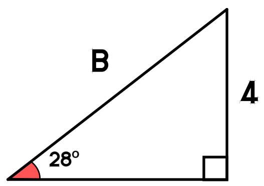
$$Y = \frac{7}{\sin(58^\circ)}$$

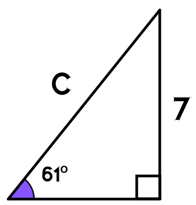
Try to find an equation that will help calculate the missing lengths in the following triangles.
Compare your strategies with a partner and discuss any similarities or differences.

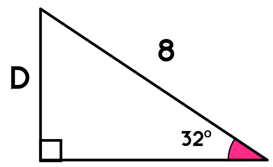


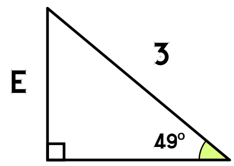
Mixed Practice

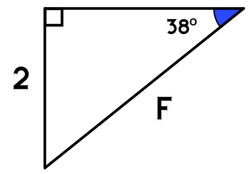


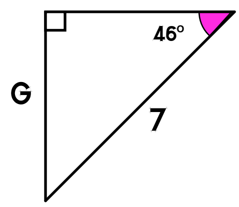


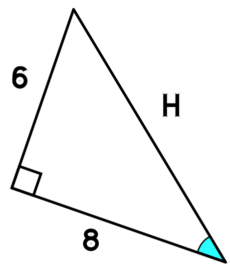


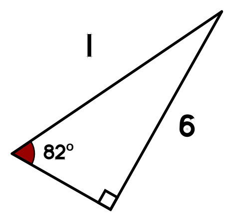


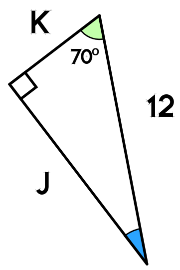


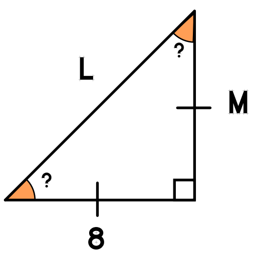


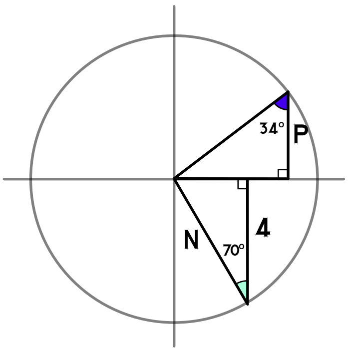


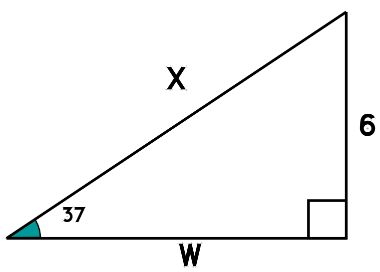


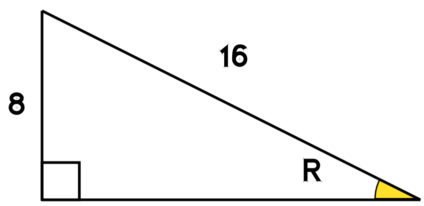












*Psst… try drawing a unit triangle and take it from there!

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